

Indian Statistical Institute Bangalore
Statistics and Mathematics Unit

Complex Analysis

M. Math. I Year – Supplementary and Back Paper

Instructor: Shubham Jain

Date: June 1, 2026

Duration: 3 Hours

Total Marks: 50

Name: _____

- Q.1.** Let Ω be a simply connected domain and $a \in \Omega$. Furthermore, let f and g are biholomorphisms such that

$$f(a) = g(a) \text{ and } f'(a) = g'(a).$$

Prove that $f = g$.

[6 Marks]

- Q.2.** Show that $\int_{-\infty}^{\infty} \frac{dx}{(1+x^2)^{n+1}} = \frac{1 \cdot 3 \cdots (2n-1)}{2 \cdot 4 \cdots (2n)} \pi$ for each $n \in \mathbb{N}$.

[6 Marks]

- Q.3.** Find $\sum_{m=1}^{\infty} \int_{|z|=1} z^m \sin\left(\frac{1}{z}\right) dz$.

[5 Marks]

- Q.4.** Let u and v be real-valued functions defined on a domain Ω . Assume that u and v are continuously differentiable and satisfy the Cauchy–Riemann equations in Ω . Show that u and v are infinitely differentiable in Ω .

[6 Marks]

- Q.5.** Prove or disprove :

There exists a power series which converges only on $\{z \in \mathbb{C} : |z| \leq 1, z \neq \pm i\}$.

[5 Marks]

- Q.6.** Consider $S_n(z) = \sum_{k=0}^n \frac{(-1)^k z^{2k}}{(2k)!}$ for $n \in \mathbb{N}$. Show that S_n has a zero in the disc with center and radius $\frac{\pi}{2}$ for infinitely many n .

[5 Marks]

- Q.7.** Does there exists an analytic function $f : \mathbb{D} \rightarrow \mathbb{D}$ with

$$f(1/2) = 3/4 \text{ and } f'(1/2) = 2/3.$$

[6 Marks]

- Q.8.** Find the number of roots of the polynomial

$$P(z) = z^{10} + 20z^7 + 10z^6 + z^5 + 3z^2 + 3z + 1$$

inside unit disc \mathbb{D} .

[5 Marks]

Q.9. Let $\mathbb{H} = \{z \in \mathbb{C} : \text{Im } z > 0\}$ and let $h : \mathbb{H} \rightarrow \mathbb{H}$ be a holomorphic map. Prove that for every $a \in \mathbb{H}$,

$$|h'(a)| \leq \frac{\text{Im } h(a)}{\text{Im } a}.$$

[6 Marks]